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Mathematical wave equation modeling in smart materials and composite continuous systems

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ABSTRAK

Pemodelan perambatan gelombang pada material cerdas dan sistem kontinu komposit menghadapi tantangan akibat interaksi multifisika, nonlokalitas bergantung skala, nonlinieritas konstitutif, dan ketidakhomogenan struktural. Tinjauan pelingkupan ini bertujuan menyintesis perkembangan pemodelan persamaan gelombang, elastodinamika, dan teknik penyelesaiannya pada periode 2019–2026 untuk aplikasi material cerdas dan sistem kontinu komposit. Berdasarkan kerangka Population–Concept–Context (PCC) dan pedoman PRISMA-ScR, artikel terverifikasi ber-DOI dikelompokkan ke dalam lima tema: struktur piezoelektrik dan magneto-electro-elastic (MEE), Functionally Graded Material (FGM) dan metamaterial, komposit berlapis dan nanostruktur nonlokal, komposit cerdas responsif medan, serta structural health monitoring (SHM) berbasis gelombang terpandu. Hasil kajian menunjukkan pergeseran dari elastodinamika linier klasik menuju pemodelan multiskala yang mengintegrasikan mikrostruktur, elastisitas permukaan, fleksoelektrik, porositas, dan nonlokalitas. FGM dan metastruktur berkembang menuju mekanisme aktif untuk band-gap tuning, konversi mode gelombang, dan pencocokan impedansi. Pada SHM, gelombang terpandu dan gelombang Lamb tetap dominan, didukung metode bebas-mesh dan machine learning. Tantangan utama meliputi variabilitas lingkungan dan standardisasi protokol validasi.

Kata Kunci: material cerdas; elastodinamika; perambatan gelombang; material komposit; metamaterial

ABSTRACT

Wave-propagation modeling in smart materials and continuous composite systems remains challenging due to multiphysics interactions, scale-dependent nonlocality, constitutive nonlinearities, and structural heterogeneity. This scoping review synthesizes recent advances in wave equations, elastodynamics, and solution techniques published during 2019–2026 for smart-material and composite-continuum applications. Using the Population–Concept–Context (PCC) framework and PRISMA-ScR guidelines, verified DOI-indexed studies were classified into five themes: piezoelectric and magneto-electro-elastic (MEE) structures, functionally graded materials (FGMs) and metamaterials, layered composites and nonlocal nanostructures, field-responsive smart composites, and guided-wave-based structural health monitoring (SHM). The findings indicate a shift from classical linear elastodynamics toward multiscale formulations incorporating microstructural length scales, surface elasticity, flexoelectricity, porosity, and nonlocal effects. FGMs and metastructures have evolved from passive stress-mitigation systems into active mechanisms for band-gap tuning, wave-mode conversion, and impedance matching. In SHM applications, guided waves and Lamb waves remain dominant, increasingly supported by mesh-free orthogonal polynomial methods and machine-learning-based feature extraction. Environmental variability and the standardization of validation protocols remain major implementation challenges.

Keywords: smart materials; elastodynamics; wave propagation; composite materials; metamaterials

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1. INTRODUCTION

The wave equation is a fundamental partial differential equation in continuum mechanics that describes how mechanical disturbances—such as displacement, stress, or strain—propagate through a medium as a function of both space and time (Othmani et al., 2022; Cawley, 2024). In its classical linear form, the equation relates the second time derivative of displacement to its spatial curvature, with the wave propagation speed determined by the elastic stiffness and mass density of the medium. When the medium is a smart material or a composite continuous system, the governing wave equation becomes substantially more complex, as additional physical fields such as electric potential, magnetic potential, and temperature distribution—must be incorporated and coupled to the mechanical displacement field through appropriate multiphysics constitutive relations (Buğday, 2026; Selvamani et al., 2023).

Smart materials are engineered to sense, actuate, or respond to environmental stimuli in a controlled and generally reversible manner (Bastola & Hossain, 2021; Kim & Zhao, 2022). Prominent examples include piezoelectric ceramics and polymers, magnetostrictive alloys, magneto-electro-elastic (MEE) composites, shape memory alloys (SMAs), and functionally graded and nanoscale materials whose mechanical and physical properties vary continuously across their geometry (Hong et al., 2022; Karami et al., 2019). Composite continuous systems consist of laminated plates, sandwich panels, shells, and fiber-reinforced beams produced by combining two or more constituent phases to achieve superior mechanical performance compared to any single constituent material alone (Si et al., 2026; Bisheh et al., 2019). Incorporating smart elements into these composite structures enables simultaneous load bearing, damage detection, energy harvesting, and active vibration attenuation, making such systems vital across aerospace, automotive, civil, biomedical, and energy engineering applications (Güemes et al., 2020; Ricci et al., 2022).

Modeling wave motion in smart and composite continuous systems is essential for three primary engineering reasons. First, designing guided-wave sensors for structural health monitoring (SHM) and nondestructive testing (NDT) requires a rigorous understanding of how phase and group velocities vary with frequency and propagation direction (Capineri & Bulletti, 2021; Philibert et al., 2022). Second, developing metamaterial resonators, shunted piezoelectric networks, and phononic structures that exploit frequency band gaps to attenuate or redirect mechanical energy demands accurate dispersion relations (Pierce et al., 2020; Moghaddaszadeh et al., 2023). Third, because modern lightweight and micro/nano-engineered structures increasingly utilize functionally graded, porous, and multiphysics materials, mathematical models must capture small-scale size effects, spatial inhomogeneities, and multi-field couplings that classical elasticity theory cannot resolve. Consequently, refined higher-order shear deformation, nonlocal elasticity, modified couple-stress, and nonlocal strain-gradient wave theories have been actively developed (Ebrahimi & Dabbagh, 2019; Wang et al., 2025).

Despite extensive research activity, the literature remains segmented into specialized subdomains: piezoelectric and magneto-electro-elastic wave theory, functionally graded material dynamics, higher-order laminated plate and shell mechanics, nonlinear wave mechanics of magnetorheological and magneto-active composites, nonlocal nanocomposite modeling, and guided-wave structural health monitoring. Each subdomain relies on distinct governing equations, solution schemes, and validation practices, yet comprehensive syntheses mapping how these mathematical formulations relate and evolve across material classes remain scarce (Othmani et al., 2022; Saber & Sedaghati, 2023). This study addresses this gap by conducting a PRISMA-ScR-guided scoping review of wave equation modeling in smart materials and composite continuous systems published between 2019 and 2026 (Tricco et al., 2018; Page et al., 2021). Specifically, the objectives of this review are to: (1) map the principal conceptual models and governing wave equations formulated during this period; (2) identify the analytical, semi-analytical, numerical, and machine-learning-assisted solution methods employed; (3) synthesize the findings across five primary classes of smart materials and composite systems; and (4) highlight persistent methodological and experimental validation gaps to guide future applied mechanics research.

2. RESEARCH METHOD

This study was conducted in accordance with the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA 2020) statement (Page et al., 2021) and its scoping review extension (PRISMA-ScR) (Tricco et al., 2018). Because a scoping review aims to map the breadth of theoretical formulations, governing equations, and solution methodologies across heterogeneous engineering systems rather than pool homogeneous quantitative effect sizes, the methodology prioritizes structured eligibility criteria, comprehensive multi-database searching, and systematic data charting.

Prior to literature retrieval, a structured review protocol was established to address the following research question: *"What conceptual models, governing wave equations, and solution methods have been reported between 2019 and 2026 for wave propagation problems in smart materials and composite continuous*

systems, and how are these mathematical frameworks distributed across the principal material classes studied in the field?" Eligibility was defined using the Population–Concept–Context (PCC) framework recommended for scoping reviews, supplemented by publication year (2019–2026), language (English), and document-type filters. The complete inclusion and exclusion criteria applied during screening are summarized in Table 1.

Table 1. Eligibility Criteria Defined Within the Population–Concept–Context (PCC) Framework

Criterion Dimension	Inclusion Criteria	Exclusion Criteria
Population / Material System	Smart materials (piezoelectric, magneto-electro-elastic, shape memory alloy, magnetostrictive, magneto-active) and composite continuous systems (laminated, sandwich, functionally graded, nanocomposite beams, plates, and shells).	Purely conventional, single-phase, non-smart isotropic materials with no composite or graded architecture.
Concept	Formulation, analytical/numerical solution, or diagnostic application of a wave equation, dispersion relation, or elastodynamic governing equation.	Studies confined to static bending, buckling, or purely thermal analysis with no wave or dynamic-propagation component.
Context	Mechanical, aerospace, civil, or biomedical engineering applications of wave modeling, including structural health monitoring and vibration/wave control.	Purely chemical synthesis or materials-processing studies with no mechanics-of-wave-propagation content.
Publication Year	2019 to 2026 inclusive.	Published before 2019 or outside the review window.
Language	English full text available.	Non-English full text with no official English translation.
Study Type	Peer-reviewed journal articles, conference papers, authoritative reviews, scholarly monographs, and indexed preprints reporting original or synthesized modeling work.	Editorials, non-peer-reviewed opinion pieces, and superseded duplicate republications.

Literature identification was executed across major academic publisher platforms and bibliographic aggregators—including ScienceDirect, SpringerLink, Taylor & Francis Online, Wiley Online Library, MDPI, IOPscience, American Chemical Society (ACS), Nature Portfolio, and the arXiv repository—combined with reference-list cross-checking and structured reference management for deduplication and thematic tagging. The search strategy combined a core phenomenon block (*wave equation*, *wave propagation*, *elastodynamics*, *dispersion*) with material-specific and application-specific blocks using Boolean operators, phrase searching, and truncation. Table 2 details the Boolean search-string architecture across the six thematic blocks. During the data-charting phase, the laminated-composite and nonlocal/strain-gradient blocks were consolidated into a unified thematic subsection due to their shared continuum and nanostructure wave formulations.

Table 2. Boolean Search-String Structure Across the Six Thematic Material and Application Blocks

Thematic Block	Boolean Search-String Structure	Applied Filters
Piezoelectric / Magneto-Electro-Elastic Composites	("wave propagation" OR "wave equation" OR dispersion) AND (piezoelectric OR "magneto-electro-elastic" OR piezomagnetic) AND (composite OR laminate)	2019–2026; English
Functionally Graded Material (FGM) Systems	("elastic wave" OR "stress wave" OR "guided wave") AND ("functionally graded" OR FGM) AND (plate OR beam OR shell)	2019–2026; English
Laminated Composite Plates and Shells	("wave propagation" OR "Lamb wave") AND (laminated OR sandwich) AND ("shear deformation theory" OR "higher-order theory")	2019–2026; English
Shape Memory Alloy / Magnetostrictive / Magneto-Active Composites	("wave propagation" OR "stress wave") AND ("shape memory alloy" OR magnetostrictive OR "magneto-active")	2019–2026; English
Nonlocal / Strain-Gradient Nanocomposites	("wave propagation" OR "wave dispersion") AND (nonlocal OR "strain gradient") AND (nanobeam OR nanoplate OR nanocomposite)	2019–2026; English
Guided-Wave Structural Health Monitoring	("guided wave" OR "Lamb wave") AND ("structural health monitoring" OR SHM) AND (composite)	2019–2026; English

Source selection followed a two-stage screening procedure. In the first stage, titles and abstracts of all retrieved records were screened against the eligibility criteria in Table 1 after removing duplicate entries. In the second stage, full-text articles of all potentially eligible records were assessed against the five verification questions presented in Table 3 to confirm the presence of an explicit wave equation, dispersion relation, or elastodynamic modeling framework, as well as verifiable DOI/bibliographic metadata. For each included study, seven data items were charted: (i) author(s) and publication year; (ii) material system investigated; (iii) mathematical theory and governing equation type; (iv) solution methodology; (v) wave or dynamic phenomenon analyzed; (vi) principal reported finding; and (vii) thematic category. Figure 1 illustrates the PRISMA 2020 flow diagram summarizing the identification, screening, eligibility assessment, and inclusion process.

Table 3. Screening Verification Template Applied at Title/Abstract and Full-Text Stages

Screening Verification Question	Inclusion Decision	Exclusion Decision
1. Is the publication year within 2019–2026?	Yes	No
2. Is the full text available in English?	Yes	No
3. Does the source concern a smart material or a composite continuous system as defined in Table 1?	Yes	No
4. Does the source formulate, solve, or apply a wave equation, dispersion relation, or elastodynamic model?	Yes	No
5. Is the study a peer-reviewed journal article, conference paper, review, monograph, or indexed preprint with verifiable bibliographic metadata?	Yes	No (editorial, opinion piece, or unverified/duplicate record)

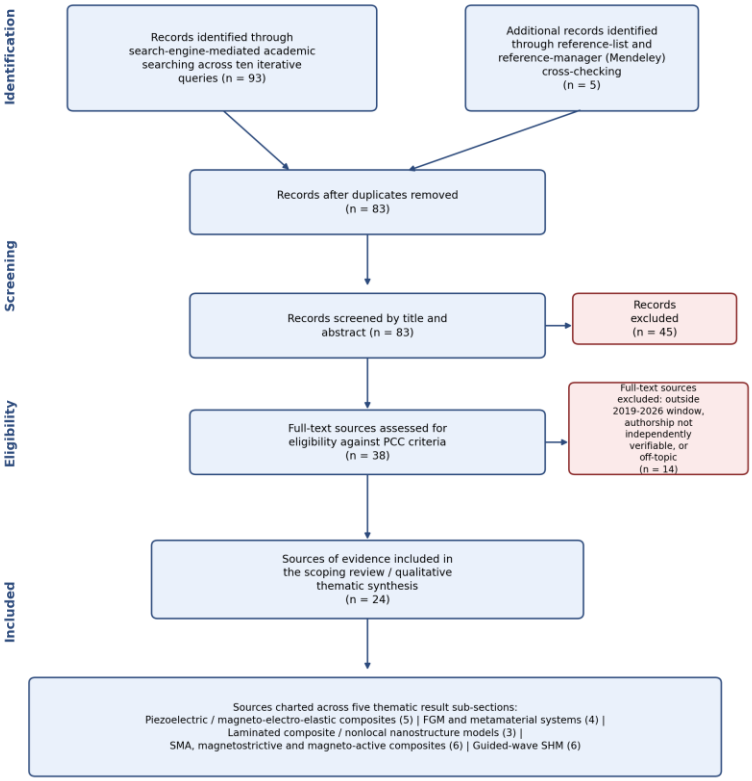


Figure 1. PRISMA 2020 Flow Diagram Summarizing the Identification, Screening, Eligibility Assessment, and Inclusion of Source.

3. RESULTS AND DISCUSSION

A. Study Selection Characteristics and Data Extraction Matrix

Following the identification, screening, and full-text eligibility assessment summarized in Figure 1, 98 records were initially identified (93 via structured database queries and 5 via citation cross-checking), yielding 83 unique records after duplicate removal. Title and abstract screening excluded 45 records, leaving 38 full-text documents for detailed eligibility evaluation. Fourteen full-text records were excluded where the study fell outside the 2019–2026 window, lacked an elastodynamic wave component, or lacked verifiable publisher metadata. Consequently, 24 primary and review sources were retained for qualitative thematic synthesis across the five inductive thematic subsections, as shown in Figure 2.

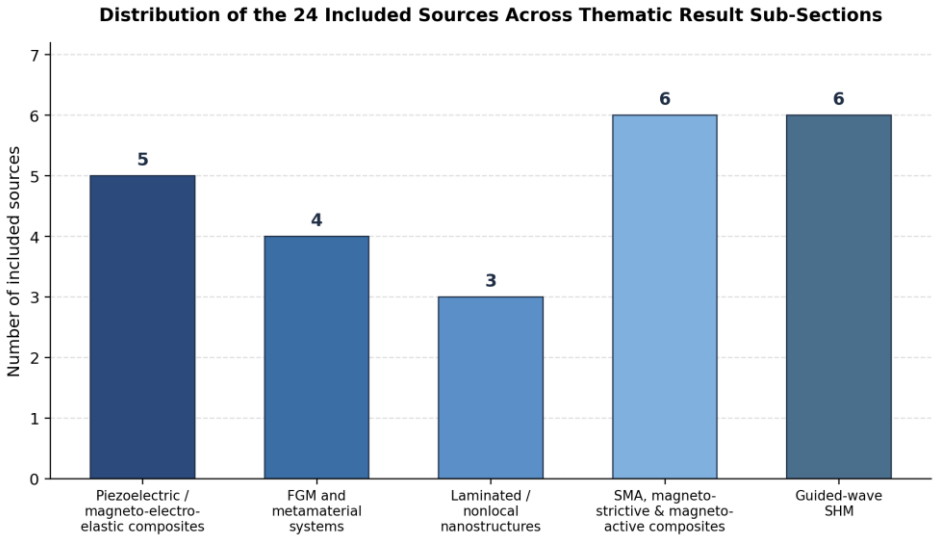


Figure 2. Distribution of the 24 Included Sources Across the Five Thematic Result Subsections

Table 4 presents the complete data extraction matrix for the 24 included sources, detailing the author(s) and publication year, the smart material or composite system examined, the governing mathematical framework, and the principal wave-propagation finding.

Table 4. Data Extraction Matrix for the 24 Sources Included in the Thematic Synthesis

Author(s) & Year	**Material System Studied**	**Theoretical / Mathematical Framework**	**Principal Wave-Propagation Finding**
Hong et al. (2022)	Magneto-electro-elastic (MEE) FG porous microbeam	Extended modified couple stress theory; Hamilton's principle	Microstructural length scale and porosity strongly shift shear, extensional, and flexural wave dispersion curves relative to classical predictions.
Wang et al. (2025)	FG piezoelectric sandwich nanoplate on viscoelastic foundation	Nonlocal strain gradient theory with surface elasticity effects	Surface elasticity, nonlocal/gradient length scales, and viscoelastic foundation damping materially alter wave dispersion in sandwich nanoplates.
Masoumi et al. (2019)	Piezoelectric nanobeam	Nonlocal strain gradient higher-order beam theory with flexoelectricity	Flexoelectric coupling produces a measurable, size-dependent shift in wave phase velocity not captured by piezoelectricity alone.
Othmani et al. (2022)	Elastic, piezoelectric, and MEE composites (review)	Orthogonal polynomial (Legendre and Jacobi) elastodynamic methods	Orthogonal polynomial expansions provide an efficient, mesh-free alternative to finite element discretization for coupled elastodynamic wave equations.
Buğday (2026)	Biocompatible foam-core sandwich nanoplate on elastic foundation	Three-dimensional thermomechanical wave formulation with magnetic field coupling	Magnetic field intensity, foam porosity distribution, and foundation stiffness jointly govern 3D thermomechanical wave phase velocities.
Karami et al. (2019)	Porous functionally graded nanoshell	Nonlocal strain gradient shell theory	Porosity coefficient and nonlocal-to-gradient length-scale ratios determine whether wave dispersion is stiffened or softened relative to classical shell theory.
Wang & Liang (2019)	Nanoporous metal foam nanobeam	Nonlocal elasticity coupled with foam micromechanics	Foam porosity distribution pattern (uniform vs. graded) alters both the cut-off frequency and group velocity of propagating flexural waves.
Cao et al. (2019)	Functionally graded viscoelastic thin film	Asymptotic wave-splitting analytical solution with numerical verification	Closed-form asymptotic Lamb-wave dispersion solutions in graded viscoelastic films agree

			closely with direct numerical simulations.
Boddapati et al. (2026); Boddapati & Daraio (2024)	2D functionally graded anisotropic metamaterial	Unit-cell Bloch dispersion analysis and frequency-domain finite element simulation	Graded density and anisotropic unit-cell geometry enable broadband longitudinal-to-shear mode conversion across stiffness-mismatched interfaces.
Selvamani et al. (2023)	Multilayered composite cylinder with Hall current and LEMV/CFRP layers	Nonlocal magneto-thermoelasticity with dual-phase-lag heat conduction	Nonlocal scale parameters and thermal phase lags jointly control the phase velocity and attenuation of magneto-thermoelastic waves.
Ebrahimi et al. (2019)	Inhomogeneous porous nanobeam	Porosity-dependent nonlocal strain gradient homogenization	A unified homogenization scheme captures the competing effects of material porosity, nonlocality, and strain gradient length scales on wave dispersion.
Ebrahimi & Dabbagh (2019); Si et al. (2026)	Smart nanostructures (monograph) and laminated sandwich structures	Nonlocal, strain-gradient, couple-stress, and stress-continuous laminate theories	Establishes unifying mathematical frameworks linking nonlocal multiphysics wave models at the nanoscale and stress-continuous laminated plates.
Saber & Sedaghati (2023)	Magnetorheological elastomers (MREs) (review)	Continuum and micromechanical field-dependent constitutive modeling	Field-dependent modulus models for MREs are well established quasi-statically but remain only partially validated in the high-frequency wave regime.
Kim & Zhao (2022)	Magnetic soft materials and soft robots (review)	Continuum magneto-mechanics	Magnetic actuation mechanisms induce strongly rate- and field-dependent stiffness that must be incorporated into dynamic wave models.
Bastola & Hossain (2021)	Magnetoactive soft composites (review)	Nonlinear magneto-elastic continuum mechanics	Shape-morphing and dynamic responses of magnetoactive composites are governed by strongly nonlinear, field-coupled constitutive behavior.
Khalid et al. (2024)	3D-printed magneto-active smart composites (review)	Multiphysics magneto-mechanical modeling	Additive manufacturing enables spatially graded magnetic filler distributions that can be tailored to program local dynamic stiffness.
Pierce et al. (2020)	Magneto-active elastomer metastructure	Bloch-wave dispersion analysis with tunable magnetic stiffness	External magnetic fields enable continuous, reversible tuning of phononic band-gap frequency ranges in elastic metastructures.
Moghaddaszadeh et al. (2023)	Particle-aligned magnetorheological metamaterial	Local-resonance band-gap continuum model	Directional alignment of magnetic particles enables anisotropic, field-tunable control over local-resonance band-gap formation.
Güemes et al. (2020)	Advanced composite structures (review)	Survey of SHM sensing technologies including guided-wave methods	Guided-wave sensing remains the most mature SHM technique for composites but requires robust baseline and environmental compensation.
Ricci et al. (2022)	Composite aerospace structures (review)	Guided (Lamb) wave propagation and damage-interaction theory	Sensor network implementation, signal processing, and damage imaging—rather than wave theory alone—are the primary bottlenecks for in-service SHM.
Capineri & Bulletti (2021)	Composite and metallic structures (review)	Ultrasonic guided-wave sensor and transducer array theory	Dense piezoelectric transducer networks enable real-time impact localization but remain sensitive to temperature variations and wave-speed anisotropy.

Philibert et al. (2022)	Aerospace composite structures (review)	Lamb-wave-based SHM technology survey	Lamb-wave SHM technologies are approaching aerospace certification readiness, though standardized validation protocols remain lacking.
Tanveer et al. (2024); Nerlikar et al. (2024)	Composite structures with guided ultrasonic wave SHM	Guided-wave dispersion theory integrated with machine learning and physics-embedded deep learning	Machine-learning feature extraction and physics-embedded deep learning frameworks increasingly complement classical guided-wave dispersion analysis.
Cawley (2024)	Long-range pipelines and plate structures (review)	Guided-wave nondestructive testing (NDT) and SHM principles	Guided-wave NDT has matured into a widely deployed industrial inspection technology, while autonomous SHM still faces field-reliability gaps.

B. Wave Equation Modeling in Piezoelectric and Magneto-Electro-Elastic Smart Composite Structures

Five included sources investigated ultrasonic and elastodynamic wave propagation in piezoelectric and magneto-electro-elastic (MEE) composite systems. Hong et al. (2022) formulated a microstructure-dependent MEE functionally graded porous Timoshenko microbeam model using the extended modified couple stress theory, deriving the coupled equations of motion via Hamilton's principle and analytically resolving the shear, extensional, and flexural wave dispersion relations. Their results confirmed that the microstructural length-scale parameter and porosity coefficient produce pronounced shifts relative to classical couple-stress-free predictions, particularly when microbeam thicknesses approach tens of micrometers.

Extending this analysis to sandwich configurations, Wang et al. (2025) examined wave propagation in a functionally graded piezoelectric sandwich nanoplate resting on a viscoelastic foundation by applying nonlocal strain gradient theory with Mori–Tanaka homogenization and surface elasticity effects. Their parametric investigation demonstrated that surface elastic constants, nonlocal and gradient length scales, and foundation viscoelastic damping all significantly shift the sandwich nanoplate's dispersion curves, with surface effects becoming increasingly dominant as plate thickness decreases. Complementing this work, Buğday (2026) modeled three-dimensional thermomechanical wave propagation in biocompatible foam-core sandwich nanoplates subjected to magnetic fields, demonstrating strong coupling between magnetic field intensity, foam porosity distribution, and foundation stiffness. Similarly, Masoumi et al. (2019) incorporated flexoelectricity into a nonlocal strain-gradient higher-order piezoelectric nanobeam model, showing that the flexoelectric coupling coefficient introduces a distinct size-dependent shift in wave phase velocity beyond piezoelectricity alone. At a methodological level, Othmani et al. (2022) reviewed orthogonal polynomial expansion methods—principally Legendre and Jacobi polynomials—and demonstrated that these mesh-free semi-analytical schemes offer substantial computational savings over finite element discretization when evaluating dispersion curves across multi-layered piezoelectric and MEE composites.

C. Wave Equation Modeling in Functionally Graded Material (FGM) and Metamaterial Systems

Four included sources focused specifically on elastic wave propagation in functionally graded material (FGM) structures and graded metamaterials. Karami et al. (2019) developed a nonlocal strain gradient shell theory for porous functionally graded nanoshells, proving that the ratio between the nonlocal parameter and the strain-gradient length-scale parameter governs whether the propagating wave experiences net stiffness hardening or softening relative to classical shell theory, while the porosity coefficient independently shifts the cut-off frequency. Similarly, Wang and Liang (2019) analyzed flexural wave propagation in nanoporous metal foam nanobeams by coupling nonlocal elasticity with a foam micromechanical model, showing that symmetric, asymmetric, and uniform through-thickness porosity distributions distinctly modify both cut-off frequencies and group velocities.

At the thin-film scale, Cao et al. (2019) derived an asymptotic wave-splitting analytical solution for Lamb-wave dispersion in functionally graded viscoelastic films and verified its accuracy against direct numerical simulations, providing a computationally efficient alternative to full finite element discretization. At the metamaterial scale, Boddapati et al. (2026) and Boddapati and Daraio (2024) combined unit-cell Bloch dispersion analysis with frequency-domain finite element simulations to design two-dimensional functionally graded anisotropic metamaterials capable of broadband longitudinal-to-shear wave mode conversion across stiffness-mismatched interfaces. Experimental validation on additively manufactured specimens confirmed that tailored spatial gradients in density and unit-cell anisotropy substantially reduce interfacial impedance mismatch and wave reflection.

D. Wave Equation Modeling in Laminated Composites and Nonlocal Nanostructure Systems

Three core sources—supported by recent laminate continuum formulations—bridged laminated composite mechanics with nonlocal and strain-gradient nanostructure theory. Selvamani et al. (2023)

investigated magneto-thermoelastic wave propagation in multilayered composite cylinders incorporating a nonlocal scale parameter and dual-phase-lag heat conduction within the coupled thermoelastic field equations. Their findings demonstrated that nonlocal scale effects and thermal relaxation phase lags jointly govern both the phase speed and attenuation of thermoelastic waves propagating through layered cylinder walls—a coupling phenomenon absent in classical Fourier thermoelasticity.

In parallel, Ebrahimi et al. (2019) formulated a porosity-dependent homogenization scheme within a nonlocal strain-gradient framework for wave dispersion analysis of inhomogeneous nanobeams, unifying material porosity with competing nonlocal softening and strain-gradient stiffening length scales. The scholarly monograph by Ebrahimi and Dabbagh (2019), *Wave Propagation Analysis of Smart Nanostructures*, synthesizes these nonlocal, strain-gradient, and couple-stress wave theories across functionally graded, piezoelectric, MEE, magnetostrictive, and porous nanobeams and nanoplates. At the macroscopic laminated-plate scale, these nanoscale formulations are complemented by higher-order and stress-continuous laminate theories (Bisheh et al., 2019; Si et al., 2026), which enforce interlaminar stress continuity across composite and sandwich interfaces without requiring prohibitive layerwise degrees of freedom.

E. Wave Equation Modeling in Shape Memory Alloy, Magnetostrictive, and Magneto-Active Smart Composites

Six included sources examined wave and dynamic modeling in shape memory alloy (SMA), magnetostrictive, and magneto-active smart composites, where the elastodynamic wave operator is coupled to strongly nonlinear, hysteretic, and field-dependent constitutive laws. Saber and Sedaghati (2023) reviewed continuum and micromechanical constitutive models for magnetorheological elastomers (MREs), observing that although field-dependent shear modulus formulations have matured significantly under quasi-static and low-frequency oscillatory loading, experimental validation in the high-frequency dynamic wave regime remains sparse. Kim and Zhao (2022) and Bastola and Hossain (2021) similarly emphasized that magnetic actuation and shape-morphing in magneto-active soft composites induce strongly rate- and field-dependent stiffness variations that must be embedded into elastodynamic operators for predictive modeling. Furthermore, Khalid et al. (2024) highlighted how additive manufacturing (3D printing) enables spatially graded magnetic particle distributions, allowing local dynamic stiffness and wave propagation pathways to be programmed during fabrication.

At the metastructure level, Pierce et al. (2020) applied Bloch-wave dispersion analysis to demonstrate that an external magnetic field can continuously and reversibly tune the frequency range of phononic band gaps in magneto-active elastomer metastructures. Extending this concept, Moghaddaszadeh et al. (2023) developed a local-resonance band-gap continuum model showing that aligning magnetic particles into chains within a magnetorheological metamaterial enables directional, field-tunable control over local-resonance attenuation zones. Collectively, these studies demonstrate that the primary modeling challenge for field-active smart composites lies in accurately resolving the nonlinear, field- and history-dependent effective modulus entering the wave operator.

F. Wave-Based Structural Health Monitoring (SHM) in Smart Composite Systems

Six included sources addressed the diagnostic application of wave equation modeling, specifically guided ultrasonic (Lamb) wave propagation for structural health monitoring (SHM) of composite structures. Güemes et al. (2020) and Ricci et al. (2022) concluded that guided-wave sensing remains the most mature SHM methodology for advanced aerospace composites; however, the primary barrier to widespread in-service adoption has shifted from fundamental wave propagation theory to practical implementation challenges, including sensor network optimization, environmental temperature compensation, and automated damage-imaging algorithms. Capineri and Bulletti (2021) showed that integrated piezoelectric transducer arrays enable real-time impact detection and localization on composite panels, provided that directional wave-speed anisotropy and environmental shifts are accurately compensated.

Surveying Lamb-wave SHM for aircraft composite structures, Philibert et al. (2022) and Cawley (2024) noted that while guided-wave inspection is well established in industrial nondestructive testing and approaching aerospace certification readiness, standardized cross-institutional validation protocols for autonomous monitoring remain underdeveloped. Notably, Tanveer et al. (2024) and Nerlikar et al. (2024) documented a major methodological transition toward hybridizing classical guided-wave dispersion models with machine-learning feature extraction and physics-embedded deep-learning frameworks, enabling rapid multi-fidelity forward and inverse wave modeling across complex composite geometries.

G. Comparative Synthesis Matrix and Cross-Cutting Discussion

To compare the mathematical and computational characteristics across the five thematic areas, Table 5 presents a qualitative synthesis matrix contrasting the governing equation structure, typical length scale, dominant solution methodology, and relative computational cost.

Table 5. Qualitative Comparative Synthesis Matrix Across the Five Thematic Result Subsections

Thematic Material / Application Class	Governing Equation Structure and Order	Typical Length Scale	Dominant Solution Methodology	Relative Computational Cost
Piezoelectric / MEE Composites	Coupled elastic–electric–magnetic PDE system, typically 4th-order with couple-stress, flexoelectric, or strain-gradient terms.	Micro- to nanoscale (microbeams, sandwich nanoplates).	Semi-analytical (Hamilton's principle) and mesh-free orthogonal polynomial expansions.	Low to moderate
FGM and Metamaterial Systems	Spatially varying-coefficient elastodynamic PDE; Bloch-periodic eigenvalue problem for metamaterials.	Meso- to macroscale (plates, shells, thin films, unit cells).	Asymptotic wave-splitting analytical solutions and finite element Bloch-mode analysis.	Moderate
Laminated / Nonlocal Nanostructures	Coupled thermo-nonlocal-elastic PDE with phase-lag thermal or strain-gradient length-scale corrections.	Nanoscale beams/shells to layered mesoscale cylinders.	Nonlocal strain-gradient analytical formulations and stress-continuous laminate models.	Moderate to high
SMA / Magnetostrictive / Magneto-Active Composites	Nonlinear, field- and history-dependent constitutive coupling embedded within an elastodynamic wave operator.	Meso- to macroscale (elastomer metastructures, 3D-printed composites).	Micromechanical/continuum constitutive modeling combined with Bloch dispersion analysis.	High (nonlinear, field-coupled)
Guided-Wave Composite SHM	Rayleigh–Lamb dispersion relations applied diagnostically (inverse problem) to damaged composite structures.	Macroscale plates, shells, aircraft panels, and pipelines.	Semi-analytical dispersion solutions combined with signal processing and physics-embedded deep learning.	Low (forward theory) to high (full sensor-network inversion)

As synthesized in Table 5, three overarching patterns emerge across the 2019–2026 literature. First, the classical constant-coefficient elastodynamic wave equation is being systematically extended across all material classes to incorporate coupled multiphysics potentials, internal microstructural length scales, spatially graded or Bloch-periodic moduli, nonlinear field-dependent hysteresis, and diagnostic inversion. Second, a methodological divide persists between micro/nanoscale studies—which predominantly employ closed-form or semi-analytical series solutions—and macroscale magneto-active or SHM studies, which rely heavily on finite element discretization, signal processing, and machine learning. Third, theoretical dispersion formulations have outpaced high-frequency dynamic experimental validation, particularly for field-responsive elastomers and nonlocal nanostructures (Saber & Sedaghati, 2023; Boddapati et al., 2026).

4. CONCLUSION

This PRISMA-ScR-guided scoping review mapped the conceptual frameworks, governing wave equations, and solution methodologies reported between 2019 and 2026 for wave propagation in smart materials and composite continuous systems. Across the 24 synthesized sources spanning five thematic domains, classical linear elastodynamics continues to serve as the foundational mathematical framework, but it has been substantially extended in domain-specific directions: (1) coupled electric-magnetic fields, porosity, surface elasticity, and flexoelectricity in piezoelectric and MEE composites; (2) spatially graded and Bloch-periodic stiffness profiles for wave-mode conversion and band-gap tuning in FGMs and metamaterials; (3) nonlocal, strain-gradient, and dual-phase-lag thermoelastic couplings in nanostructures and laminated cylinders; (4) nonlinear, rate- and field-dependent constitutive laws in SMA, magnetostrictive, and 3D-printed

magneto-active composites; and (5) diagnostic inversion of Lamb-wave dispersion relations increasingly accelerated by physics-embedded deep learning in composite SHM.

Based on the research gaps identified in this synthesis, four strategic recommendations are proposed to guide future applied mechanics research and engineering practice: (1) developing unified multiscale computational solvers capable of bridging nanoscale semi-analytical formulations and macroscale finite element models within a single architecture; (2) conducting systematic high-frequency, wave-regime experimental testing to validate nonlinear constitutive models for magnetorheological and magneto-active composites; (3) establishing standardized validation and environmental-compensation protocols to accelerate aerospace and industrial certification of guided-wave SHM systems; and (4) advancing hybrid physics-embedded machine-learning frameworks that combine first-principles elastodynamic wave mechanics with data-driven damage inference. A summary of these recommendations, their target systems, and relevant stakeholders is presented in Table 6.

Table 6. Summary of Strategic Recommendations for Wave Equation Modeling in Smart Materials and Composites

No.	Strategic Recommendation	Target System / Methodology	Relevant Stakeholders
1	Develop unified multiscale computational solvers linking semi-analytical nanoscale models with macroscale finite/spectral element architectures.	Cross-scale wave modeling from piezoelectric nanobeams/nanoplates to macroscopic laminated composite panels.	Computational mechanics researchers and scientific software developers.
2	Expand high-frequency, wave-regime experimental characterization beyond quasi-static testing.	Nonlinear, field-dependent constitutive models for MREs, SMAs, and 3D-printed magneto-active composites.	Experimental solid mechanics laboratories and smart-materials engineers.
3	Establish standardized validation protocols and robust environmental/temperature compensation pipelines.	Lamb-wave and guided ultrasonic wave structural health monitoring (SHM) networks in composite structures.	Aerospace/civil engineering industries, certification bodies, and NDT/SHM practitioners.
4	Integrate first-principles elastodynamic PDEs with physics-embedded deep learning and surrogate modeling.	Real-time dispersion inversion, impact localization, and automated damage classification in composites.	Applied mathematicians, data scientists, and structural diagnostics engineers.

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