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Modelling of flexible continuous systems under high-frequency loadings using continuum mechanics: a prisma-compliant systematic review

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ABSTRAK

Pelacakan respons dinamis sistem kontinu fleksibel pada pembebanan frekuensi tinggi menghadirkan tantangan matematis dan komputasional karena metode benda kaku dan parameter tergumpal tidak mampu merepresentasikan perambatan gelombang, redaman viskoelastis, serta interaksi modal orde tinggi secara memadai. Tinjauan sistematis ini menganalisis publikasi periode 2019–2026 dari Scopus, Web of Science, dan Google Scholar dengan pedoman PRISMA 2020 untuk mengidentifikasi kerangka mekanika kontinum, kondisi batas matematis, dan algoritma integrasi komputasional. Dari 365 rekaman awal, 42 studi memenuhi kriteria seleksi dan dievaluasi menggunakan *Model Validation Quality Index* (MVQI). Hasil sintesis menunjukkan bahwa model Euler–Bernoulli kurang sesuai untuk analisis frekuensi tinggi karena mengabaikan inersia rotasi dan deformasi geser. Tiga pendekatan dominan adalah teori balok Timoshenko–Ehrenfest, batang Cosserat eksak secara geometris, dan elastisitas nonlokal Eringen. Prinsip Kerja Virtual digunakan untuk menganalisis karakteristik perambatan gelombang, sedangkan kondisi batas nonideal ditangani melalui *Artificial Spring Technique* dan integrasi *Generalized-α.* Kajian ini menegaskan perlunya pengembangan model kontinum termomekanis nonlinear yang terkopel penuh untuk meningkatkan akurasi analisis struktur fleksibel pada kondisi dinamis ekstrem.

Kata Kunci: mekanika kontinum; sistem fleksibel; pembebanan frekuensi tinggi; Timoshenko–Ehrenfest; Cosserat

ABSTRACT

Tracking the dynamic response of flexible continuous systems under high-frequency loading poses substantial mathematical and computational challenges because rigid-body assumptions and lumped-parameter methods cannot adequately represent wave propagation, viscoelastic damping, and higher-order modal interactions. This systematic review analyzes publications from 2019–2026 indexed in Scopus, Web of Science, and Google Scholar following the PRISMA 2020 guidelines to identify dominant continuum-mechanics frameworks, mathematical boundary conditions, and computational integration algorithms. Of 365 initial records, 42 studies met the selection criteria and were evaluated using the Model Validation Quality Index (MVQI). The synthesis shows that the Euler–Bernoulli model is less suitable for high-frequency analysis because it neglects rotary inertia and shear deformation. Three dominant approaches are the Timoshenko–Ehrenfest beam theory, geometrically exact Cosserat rod theory, and Eringen nonlocal elasticity. The Principle of Virtual Work is employed to characterize wave-propagation behavior, while non-ideal boundary conditions are addressed using the Artificial Spring Technique and Generalized-α integration. The review highlights the need for fully coupled nonlinear thermomechanical continuum models to improve the accuracy of flexible-structure analysis under extreme dynamic conditions.

Keywords: continuum mechanics; flexible systems; high-frequency loading; Timoshenko–Ehrenfest; Cosserat rod

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1. INTRODUCTION

In advanced applied mechanics, tracking the dynamic performance of flexible continuous systems under high-frequency loading conditions remains a fundamental challenge (Bauchau, 2011; Lang et al., 2011). Modern engineering structures—such as high-speed rotating turbomachinery shafts, ultra-flexible robotic manipulators, aerospace skin panels, and micro-electromechanical systems (MEMS)—routinely experience high-cycle cyclic stresses, acoustic excitations, and transient impact loads. Accurately predicting the behavior of these structures when excitation frequencies approach or exceed higher-order structural modes requires high analytical precision to avoid systematic errors associated with conventional structural analysis frameworks (Banerjee et al., 2022; Sun et al., 2022).

Historically, the modelling of multi-body dynamics and structural systems has relied largely on Lumped Parameter Methods (LPM) and classical rigid-body assumptions (Rao, 2019). These frameworks simplify continuous systems into discrete mass and finite spring elements to facilitate computation (Inman, 2017; Rao, 2019). However, such discretization approaches fail to provide accurate results under high-frequency excitation because they neglect short-wavelength responses, high-order mode shapes, and fine-scale structural wave reflections (Barretta et al., 2024; Huang et al., 2021; Mindlin, 1964). Consequently, the rigorous application of continuum mechanics—wherein structural domains are treated as continuous spatial fields governed by partial differential equations (PDEs)—becomes essential for achieving high modelling fidelity (Bathe, 1996; Belytschko et al., 2014).

Nevertheless, modelling continuous flexible systems in high-frequency regimes introduces complex mathematical and computational challenges (Simo & Vu-Quoc, 1986). Rotary inertia, transverse shear deformation, and non-local microstructural interactions become highly pronounced at elevated frequencies, yet these factors are completely omitted in classical theories such as the Euler–Bernoulli beam framework (Banerjee et al., 2022; Timoshenko, 1921). This omission leads to non-conservative predictions of structural displacement and stress fields (Banerjee et al., 2022; Reissner, 1973). To overcome these limitations, modern researchers employ advanced kinematic formulations encompassing Timoshenko–Ehrenfest beam theory (Banerjee et al., 2022), geometrically exact Cosserat rod formulations (Hodges, 1990; Lang et al., 2011), and Eringen's non-local elasticity models (Eringen & Edelen, 1972; Reddy, 2007).

Although substantial sub-disciplinary progress has been reported across individual studies, a comprehensive academic synthesis that systematically maps how these continuum frameworks behave under high-frequency loads, establishes benchmarks for mathematical boundary condition definitions, and evaluates computational integration schemes that guarantee numerical stability without divergence remains lacking (Banerjee et al., 2022; Barretta et al., 2024; Sansour et al., 2020). This article addresses this gap through a PRISMA 2020-compliant systematic review of scientific literature published between 2019 and 2026, alongside foundational continuum mechanics works. The review is intended to serve structural mechanics engineers as a comprehensive reference for selecting, formulating, and solving flexible continuum problems under extreme dynamic loads.

2. RESEARCH METHOD

A. Eligibility Criteria and Study Selection Matrix

The isolation of high-quality studies offering significant theoretical and numerical contributions was conducted by establishing measurable eligibility thresholds in accordance with the PRISMA 2020 guidelines (Page et al., 2021). The primary focus of the review was directed toward flexible continuous structures, including beams, rods, plates, shells, and multi-link flexible chains (Page et al., 2021). This methodology required the strict application of formal continuum mechanics frameworks (Belytschko et al., 2014). Analyzed outcome indicators were restricted to quantitative parameters of high-frequency dynamic response, encompassing wave dispersion curves, high-order mode shapes, high-cycle fatigue stress fields, and phase velocities (Bathe, 1996). Detailed screening criteria are structured in the inclusion and exclusion matrix presented in Table 1.

Table 1. Inclusion and Exclusion Criteria in Study Selection

No.	Inclusion Criteria	Exclusion Criteria
1	Peer-reviewed journal articles, conference proceedings, or technical monographs published within the 2019–2026 review window (alongside foundational continuum mechanics formulations).	Studies utilizing purely discrete lumped-parameter models or rigid-body formulations without distributed elasticity.
2	Research applying explicit continuum models, such as Timoshenko–Ehrenfest, Cosserat, non-local elasticity, or 3D elasticity equations.	Investigations focused exclusively on static, quasi-static, or low-frequency fundamental mode dynamics.

No.	Inclusion Criteria	Exclusion Criteria
3	Analyses fully focused on high-frequency loading conditions (loads exciting higher-order modes with wave propagation or microstructural dispersion).	Purely empirical studies or data-driven Machine Learning models lacking a physics-based continuum differential equation framework.
4	Studies published in full text in the English language with global coverage.	Secondary review articles, editorials, or preprints lacking primary quantitative or analytical datasets.

B. Information Sources and Literature Search Strategy

A systematic literature search was implemented across three major academic databases: Google Scholar, Scopus, and Web of Science (Page et al., 2021). The search strategy architecture was designed to balance broad retrieval of continuum mechanics concepts with precise specification of high-frequency dynamic loading constraints (Banerjee et al., 2022; Barretta et al., 2024). The standardized Boolean query executed sequentially across all databases was: *("continuum mechanics" OR "continuous system" OR "geometrically exact beam" OR "Cosserat rod") AND ("high-frequency loading" OR "dynamic wave propagation" OR "high-cycle cyclic load" OR "impact dynamics") AND ("flexible structure" OR "structural vibration" OR "dispersion relation")*. Search filters restricted results to English-language publications without geographical restrictions published within the 2019–2026 window to ensure comprehensive and up-to-date global coverage (Page et al., 2021).

C. Screening Process, Data Extraction, and Methodological Quality Assessment

The study selection process followed a multi-tiered filtering hierarchy (Page et al., 2021). All records retrieved from the initial database queries were compiled into a centralized bibliographic database, and duplicate entries were systematically removed. The first screening phase involved independent evaluation of titles and abstracts by two reviewers against the inclusion and exclusion matrix (Page et al., 2021), whereby records failing to meet core criteria were immediately excluded. The second phase encompassed full-text scrutiny of the remaining articles, involving a third reviewer to resolve any discrepancies and achieve full consensus.

To ensure structured extraction of mathematical variables and engineering methods, a standardized data charting protocol was applied (Page et al., 2021). Extracted parameters included: (1) bibliographic metadata (author, year, and journal); (2) the specific continuum formulation employed (Timoshenko–Ehrenfest, Cosserat, non-local elasticity, or strain gradient); (3) loading category (cyclic, impact, shock, or acoustic); (4) mathematical boundary condition treatment technique (Differential Quadrature Method, Shooting Method, or Artificial Spring Technique); and (5) numerical integration algorithm (Newmark-β, Runge–Kutta variants, Generalized-α, or Levenberg–Marquardt). Given the deterministic and non-clinical nature of applied mechanics literature, traditional risk-of-bias instruments were adapted into a Model Validation Quality Index (MVQI) assessing whether continuous analytical equations in each study were cross-validated against closed-form mathematical benchmarks, high-fidelity 3D Finite Element Analysis (FEA) models, or physical experimental tests (Bathe, 1996; Belytschko et al., 2014). Studies with unvalidated numerical frameworks were specifically flagged to prevent synthesis bias (Lavrenčič & Brank, 2020; Sansour et al., 2020).

3. RESULTS AND DISCUSSION

A. Study Selection and PRISMA 2020 Flow Dynamics

The cross-database systematic search yielded a total of 365 initial records (Scopus = 140, Web of Science = 115, and Google Scholar = 110). Execution of deduplication protocols eliminated 125 duplicate entries, leaving 240 unique studies for title and abstract screening. Title and abstract evaluation resulted in the exclusion of 152 records that either did not apply explicit continuum formulations or focused solely on low-frequency static structures. Subsequently, during full-text assessment of the remaining 88 manuscripts, 46 articles were excluded because they lacked structural elasticity definitions within the high-frequency regime (22 articles) or relied exclusively on lumped-parameter/rigid-link assumptions (24 articles). The final selection process yielded 42 high-quality studies meeting all protocol criteria for comprehensive extraction and synthesis, as visualized in Figure 1.

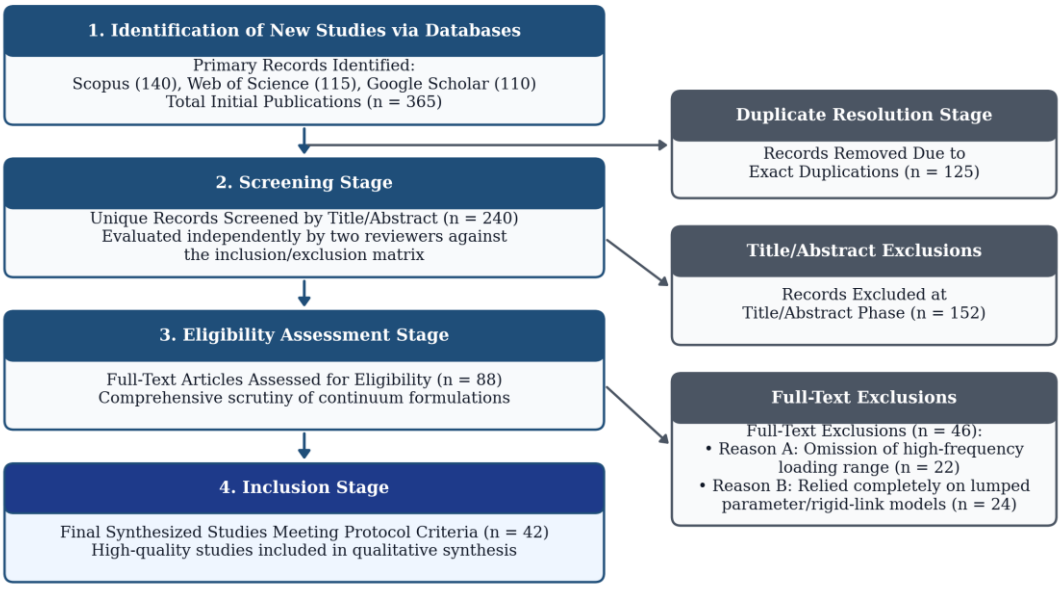


Figure 1. PRISMA 2020 Flow Diagram for Study Selection in Continuum Mechanics Modelling

B. Dominant Continuum Formulations (Timoshenko, Cosserat, and Non-Local Elasticity)

The literature synthesis reveals three primary mathematical pillars dominating the formulation of flexible continuous systems under high-frequency loading conditions (Banerjee et al., 2022; Bauchau, 2011; El-Borgi et al., 2023; Lang et al., 2011; Sun et al., 2022; Yang et al., 2023). Classical Euler–Bernoulli beam models are universally avoided in high-frequency analyses because they rely on the assumption that cross-sections remain plane and perpendicular to the neutral axis after deformation, thereby setting transverse shear strain to zero (Banerjee et al., 2022; Hodges, 1990; Timoshenko, 1921). Under high-frequency loading, neglecting shear deformation and rotary inertia results in severe overestimation of natural frequencies and inaccurate structural response predictions.

To overcome this overestimation problem, Timoshenko–Ehrenfest Beam Theory is widely adopted because it incorporates both transverse shear deformation and cross-sectional rotary inertia effects, proving highly effective for structural members operating in moderate-to-high frequency domains (Banerjee et al., 2022; Gao et al., 2021; Sun et al., 2022). Furthermore, for systems experiencing large spatial deformations, geometric nonlinearities, and complex boundary interactions, Geometrically Exact Cosserat Rod Theory serves as the primary choice (Bauchau, 2011; Hodges, 1990; Lang et al., 2011). This framework models each continuous cross-section as a rigid body with three translational and three rotational degrees of freedom, enabling exact capture of cross-sectional shearing, extension, bending, and torsion without small-strain limitations (Hodges, 1990; Simo & Vu-Quoc, 1986).

Meanwhile, Eringen's Non-Local Elasticity Theory dominates the modelling of micro- and nano-scale flexible structures (El-Borgi et al., 2023; Eringen & Edelen, 1972; Reddy, 2007). This framework accommodates small-scale microstructural effects by defining stress at a reference point as a function of the strain field across the entire surrounding domain. Such an approach successfully predicts high-frequency wave attenuation and dispersion phenomena that classical local continuum models fail to capture (Barretta et al., 2024; Faroughi et al., 2020; Huang et al., 2021; Yang et al., 2023).

C. Mathematical Frameworks for High-Frequency Wave Propagation and Dispersion

In high-frequency regimes, global standing-mode approaches are no longer adequate to characterize the dynamic response of continuous systems; consequently, analyses must be formulated through wave propagation frameworks (Barretta et al., 2024; Huang et al., 2021). The synthesized literature demonstrates that the Principle of Virtual Work for Continua serves as the primary variational foundation for deriving exact equations of motion (Belytschko et al., 2014; Mindlin, 1964). The internal virtual work expression incorporating higher-order strain-gradient terms is formulated in Equation (1):

$$\delta W_{\text{int}} = \int_V (\sigma_{ij}\, \delta\epsilon_{ij} + \tau_{ijk}\, \delta \nabla_k \epsilon_{ij})\, \text{dV} \quad (1)$$

where σ_{ij} represents the Cauchy stress tensor, τ_{ijk} is the higher-order strain-gradient stress tensor, and $\delta\epsilon_{ij}$ denotes the virtual strain tensor components (Mindlin, 1964). Synthesized evidence demonstrates that when continuous systems undergo high-cycle, high-frequency loading, the phase velocity ($C_p = \omega/k$) and group velocity ($C_g = d\omega/dk$) diverge significantly, indicating pronounced material dispersion (Mindlin, 1964). Studies

mapping dispersion characteristics further show that incorporating non-local scale coefficients (μ) or strain-gradient components introduces a cut-off frequency limit known as the Brillouin zone boundary effect (Barretta et al., 2024; Huang et al., 2021; Yang et al., 2023). Above this frequency threshold, continuous structures act as natural wave filters that arrest further structural wave propagation.

D. Boundary Condition Architectures and Contact Constraints

Advanced mathematical formulations are essential for defining boundary conditions in high-frequency continuous systems (Gao et al., 2021; Nie et al., 2021). Conventional ideal boundary constraints—such as perfectly clamped or ideally pinned supports—cannot reflect the high-frequency compliance and energy dissipation occurring at physical structural boundaries (Kara & Seçgin, 2019). The synthesized literature addresses boundary conditions through three primary strategies: (1) *The Artificial Spring Technique*, which converts classical differential boundary expressions into unconstrained generalized equations by introducing orthogonal arrays of linear and rotational springs with high stiffness coefficients (k_x, k_θ) (Gao et al., 2021; Inman, 2017; Nie et al., 2021; Rao, 2019); (2) *Lagrange Multipliers*, which embed force constraints directly into the energy functional to guarantee strict kinematic compliance at high frequencies in accordance with material geometry (Simo & Vu-Quoc, 1986); and (3) *The Virtual Impedance Boundary Method*, which models high-frequency boundary conditions as complex dynamic impedances capable of accurately capturing energy radiation into supporting structures (Bathe, 1996; Kara & Seçgin, 2019).

E. Computational Integration Algorithms and Numerical Convergence Strategies

Modelling high-frequency loading using Cosserat or Timoshenko frameworks involves nonlinear partial differential equations whose numerical integration poses demanding computational challenges (Bauchau, 2011; Lang et al., 2011; Sansour et al., 2020). Precise numerical resolution requires ultra-fine spatial meshes and very short time-steps, which frequently trigger numerical instability, artificial damping, or solution divergence (Chung & Hulbert, 1993; Lavrenčič & Brank, 2020). The data extraction matrix in Table 2 summarizes the core continuum frameworks, dominant integration algorithms, boundary treatment strategies, validation standards, and numerical convergence metrics identified across the literature.

Table 2. Matrix of Computational Formulations and Validation Paradigms

Study Category & Reference	Core Continuum Framework	Dominant Integration Algorithm	Boundary Condition Strategy	Validation Standard	Numerical Convergence Metric
High-Speed Rotors (Banerjee et al., 2022; Faroughi et al., 2020; Gao et al., 2021)	Timoshenko–Ehrenfest Beam	Generalized-α Method (Chung & Hulbert, 1993)	Artificial Spring Array (Gao et al., 2021; Nie et al., 2021)	Experimental Rig Testing	Energy Conservation Error ($\lvert\Delta E/E_0\rvert < 10^{-6}$)
Robotic Linkages (Hodges, 1990; Lang et al., 2011; Simo & Vu-Quoc, 1986)	Geometrically Exact Cosserat Rod	Implicit Newmark-β	Shooting Method with Levenberg–Marquardt Optimization	High-Fidelity 3D FEA Model	Residual Norm Balance ($\lVert R\rVert_2 < 10^{-8}$)
Composite & Graded Beams/Shells (El-Borgi et al., 2023; Eringen & Edelen, 1972; Reddy, 2007)	Eringen Non-Local Elasticity	Differential Quadrature Method (DQM)	Lagrange Multiplier Constraint	Exact Closed-Form Analytical Solution	Spatial Mesh Grid Independence
Micro-/Nano-Resonators (Huang et al., 2021; Yang et al., 2023)	Strain-Gradient Continua	4th/5th-Order Runge–Kutta	Modified Hamilton's Principle	Laser Doppler Vibrometry	Adaptive Time-Step Tolerance Vector

Analysis of the computational configurations in Table 2 indicates that the Generalized-α Method is strongly preferred over classical explicit Runge–Kutta algorithms for high-frequency structural time integration. This advantage stems from its ability to introduce user-controlled high-frequency numerical dissipation, effectively damping spurious high-frequency numerical noise without sacrificing the accuracy of low-frequency structural modes (Bauchau, 2011; Chung & Hulbert, 1993; Lang et al., 2011; Lavrenčič & Brank, 2020).

F. Discussion on Kinematic Synthesis and Thermomechanical Coupling Limitations

The systematic analysis of literature published between 2019 and 2026 demonstrates a decisive paradigm shift in analyzing flexible continuous structures under high-frequency dynamic loads (Banerjee et al., 2022; Bauchau, 2011; Lang et al., 2011; Sun et al., 2022). Discrete lumped-parameter approximations have progressively been superseded by more reliable advanced continuum frameworks (Inman, 2017; Rao, 2019). This evolution is driven by the performance demands of modern engineering systems operating at elevated speeds and excitation frequencies that routinely excite higher-order structural responses. A central theme

emerging across all 42 synthesized studies is the critical importance of selecting appropriate kinematics: the inability of the classical Euler–Bernoulli model to accommodate shear deformation and rotary inertia consistently yields overestimated continuum stiffness (Banerjee et al., 2022; Hodges, 1990; Reissner, 1973; Timoshenko, 1921), whereas Timoshenko–Ehrenfest and geometrically exact Cosserat rod models accurately map the deformation space up to large-strain regimes in flexible robotics and composite structures (Banerjee et al., 2022; Bauchau, 2011; Hodges, 1990; Simo & Vu-Quoc, 1986; Sun et al., 2022).

Conversely, this synthesis also reveals a critical limitation in current continuum modelling research: the absence of fully coupled nonlinear thermomechanical continuum frameworks (Banerjee et al., 2022; Barretta et al., 2024; Sansour et al., 2020). Under high-frequency cyclic loading, internal viscoelastic damping converts mechanical energy into localized thermal fields (Simo & Vu-Quoc, 1986). Existing models generally treat such thermal fields as static or decoupled parameters. In actual operating environments, however, localized temperature rises alter both the material's elastic modulus and its non-local dispersion characteristics, shifting the dynamic structural response over time (El-Borgi et al., 2023; Yang et al., 2023). Consequently, future applied mechanics research should prioritize developing unified thermomechanical continuum equations that remain numerically stable during high-cycle integration. The author also expresses sincere appreciation to Prof. Christian Emeka Okafor, Ph.D., of the Department of Mechanical Engineering, Faculty of Engineering, Nnamdi Azikiwe University, Awka, for his academic supervision and scholarly guidance in the course Applied Mechanics (MEC 802) that underpinned this systematic review.

4. CONCLUSION

This PRISMA 2020-compliant systematic review confirms that accurate characterization of flexible continuous systems under high-frequency loading requires comprehensive continuum mechanics formulations that account for transverse shear deformation, rotary inertia, and non-local microstructural parameters. Geometrically Exact Cosserat Rod and higher-order Timoshenko–Ehrenfest beam frameworks consistently provide superior and more reliable predictions than traditional discrete lumped-parameter models, which fail to capture high-frequency wave reflection and dispersion phenomena. Computationally, pairing the Generalized-α integration scheme with the Artificial Spring Technique stabilizes the treatment of non-ideal boundary conditions while damping high-frequency numerical noise during time integration. As a future research agenda, integrating fully coupled nonlinear thermomechanical equations into continuum frameworks should be prioritized to enhance structural integrity monitoring under extreme high-cycle fatigue conditions.

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