Analytical methods and recent advances in computation for the dynamics of continuous systems

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ABSTRAK

Sistem kontinu seperti balok, pelat, cangkang, dan membran merupakan sistem berparameter terdistribusi yang perilaku dinamisnya diatur oleh persamaan diferensial parsial. Analisis sistem tersebut penting untuk memprediksi getaran, resonansi, stabilitas, dan respons transien, namun solusi eksak sering sulit diperoleh akibat kompleksitas geometri, nonlinieritas material, dan kondisi batas. Penelitian ini meninjau metode analitis dan perkembangan komputasi terkini dalam dinamika sistem kontinu berdasarkan kerangka PRISMA 2020. Sebanyak 58 studi periode 2018–2026 dianalisis, mencakup teknik analitis dan semi-analitis, metode diskretisasi numerik, metode elemen spektral, *isogeometric analysis*, *model-order reduction*, *physics-informed machine learning*, dan *neural operators*. Hasil kajian menunjukkan bahwa solusi analitis klasik dan metode perturbasi tetap penting sebagai tolok ukur verifikasi dan analisis resonansi nonlinier. Metode elemen hingga dan formulasi orde tinggi mampu menangani geometri kompleks secara akurat, sedangkan reduksi orde model berbasis *spectral submanifolds* dan manifol kuadratik secara signifikan menekan biaya komputasi. Pendekatan *scientific machine learning* dan *neural operators* menawarkan prediksi cepat, tetapi keandalannya bergantung pada keterwakilan data pelatihan dan konsistensi fisika di luar rezim pelatihan.

Kata Kunci: sistem kontinu; dinamika struktur; metode elemen hingga; reduksi orde model; scientific machine learning

ABSTRACT

Continuous systems such as beams, plates, shells, and membranes are distributed-parameter systems whose dynamic behavior is governed by partial differential equations. Their analysis is essential for predicting vibration, resonance, stability, and transient structural responses, yet exact solutions are often difficult to obtain because of complex geometries, material nonlinearities, and boundary conditions. This study reviews analytical methods and recent computational advances in continuous-system dynamics using the PRISMA 2020 framework. A total of 58 studies published between 2018 and 2026 were analyzed, covering analytical and semi-analytical techniques, conventional numerical discretization, spectral element methods, isogeometric analysis, model-order reduction, physics-informed machine learning, and neural operators. The findings show that classical analytical solutions and perturbation methods remain essential as verification benchmarks and for nonlinear resonance analysis. Finite element and high-order structure-preserving formulations effectively handle complex geometries, while nonlinear model reduction using spectral submanifolds and quadratic manifolds substantially reduces computational cost. Scientific machine learning and neural operators enable rapid prediction for parametric differential-equation families, although their reliability strongly depends on training-data representativeness and physical consistency outside the training regime.

Keywords: continuous systems; structural dynamics; finite element method; model-order reduction; scientific machine learning

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1. INTRODUCTION

Continuous systems constitute a fundamental class of engineering systems in which the physical field variables of interest—such as displacement, velocity, temperature, or pressure—vary continuously over both space and time. Unlike discrete systems, which can be represented by a finite number of degrees of freedom, continuous systems possess theoretically infinite degrees of freedom and are mathematically governed by partial differential equations (PDEs) or coupled ordinary–partial differential systems. In mechanical, aerospace, and civil engineering, beams, plates, membranes, shells, and related distributed-parameter components are primary structural elements whose dynamic behavior is critical because vibration, resonance, dynamic instability, and nonlinear response directly govern structural integrity, reliability, and service life (Pellicano et al., 2019; Ravichandran et al., 2026).

The mathematical formulation of continuous-system dynamics originates from conservation laws—such as linear and angular momentum balance or Hamilton's variational principle—combined with appropriate kinematic and constitutive relations. Depending on the structural geometry and material assumptions, the governing equations yield spatial–temporal partial differential equations. For example, the transverse free vibration of an idealized, uniform Euler–Bernoulli beam is governed by the fourth-order partial differential equation:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0, \quad (1)$$

where $w(x,t)$ denotes the transverse displacement at spatial coordinate x and time t, E is Young's modulus, I is the second moment of area of the cross-section, ρ is the mass density, and A is the cross-sectional area. Solving this class of governing equations subject to appropriate initial and boundary conditions yields essential dynamic characteristics, including natural frequencies, eigenmodes, and transient or steady-state responses. Analogous higher-order partial differential operators govern the dynamics of plates, shells, membranes, acoustic cavities, and fluid–structure interaction systems (Harik et al., 1992; Yessenbayeva et al., 2018; Xing et al., 2022).

Classical analytical approaches—including separation of variables, eigenvalue analysis, modal superposition, integral transforms, and Green's function formulations—provide exact closed-form solutions for idealized configurations, offering transparent physical insight into parameter sensitivity and serving as rigorous verification benchmarks for numerical solvers (Xing et al., 2022). However, exact analytical tractability rapidly breaks down when confronted with realistic engineering complexities such as irregular geometries, spatially heterogeneous or hyperelastic materials, geometric and contact nonlinearities, non-classical boundary conditions, or multi-field couplings. To bridge this gap, approximate analytical and semi-analytical methods—such as the Rayleigh–Ritz and Galerkin projections, perturbation techniques, the method of multiple scales, and averaging approaches—reduce infinite-dimensional PDEs into tractable finite-dimensional systems while preserving key nonlinear features such as amplitude–frequency backbone curves, internal resonances, and bifurcation phenomena (Meesala & Hajj, 2019; H.-Y. Chen et al., 2020).

For general engineering structures with arbitrary geometry and complex constitutive laws, numerical discretization methods—namely the finite element method (FEM), finite difference method (FDM), finite volume method (FVM), and boundary element method (BEM)—have become the standard computational workhorses, complemented by time-integration and discrete-event quantized state system schemes (Jacinto Portela et al., 2022; Fu et al., 2023; Castro et al., 2024). Nevertheless, as spatial resolution increases and long-time or multi-query simulations are required, conventional low-order discretizations incur prohibitive computational costs. This limitation has driven three major waves of computational advancement: (1) high-order and geometry-accurate discretizations, including spectral element methods, hybridized discontinuous Galerkin formulations, and isogeometric analysis (IGA) (Buffa et al., 2022; Hafeez & Krawczuk, 2023; Takacs, 2023); (2) linear and nonlinear model-order reduction (MOR) techniques, ranging from component mode synthesis and proper orthogonal decomposition (POD) to spectral submanifolds (SSMs) and quadratic manifolds (Jain & Haller, 2022; Geelen et al., 2023; Agathos et al., 2024); and (3) scientific machine learning (SciML), physics-informed neural networks (PINNs), and discretization-invariant neural operators that learn solution mappings directly between infinite-dimensional function spaces (Raissi et al., 2019; L. Lu et al., 2021; Azizzadenesheli et al., 2024).

Given the rapid proliferation of these diverse paradigms, a systematic synthesis comparing their mathematical foundations, computational trade-offs, and domain applicability is needed. Accordingly, this paper conducts a PRISMA 2020-guided systematic review tracing the evolution of continuous-system dynamics modeling from classical and approximate analytical techniques to high-order numerical discretizations, nonlinear model-order reduction, physics-informed machine learning, and neural operators. The study evaluates the strengths, limitations, and computational trade-offs of each methodology and identifies critical research gaps to guide future work in computational mechanics.

2. RESEARCH METHOD

This review was designed and reported in accordance with the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA 2020) guidelines (Page et al., 2021) to ensure methodological transparency and reproducibility. The review addresses two primary research questions: (1) *What analytical and computational methods have been formulated for the dynamic analysis of continuous mechanical systems?* and (2) *How have recent advances in high-order discretization, nonlinear model-order reduction, and scientific machine learning addressed the limitations of traditional modeling approaches?* Based on these questions, the specific objectives of the study are to: (a) classify classical exact and approximate analytical methods for continuous-system dynamics; (b) evaluate conventional and structure-preserving numerical discretization methods; (c) examine high-order spectral, discontinuous Galerkin, and isogeometric formulations; (d) analyze linear and nonlinear model-order reduction frameworks, particularly spectral submanifolds and operator inference; and (e) synthesize emerging physics-informed machine learning and neural-operator paradigms along with their comparative trade-offs.

Structured eligibility criteria were established prior to literature retrieval. The inclusion criteria encompassed: (1) peer-reviewed journal articles, conference proceedings, and authoritative preprints investigating analytical, numerical, reduced-order, or data-driven methods for continuous or distributed-parameter dynamic systems (specifically beams, plates, shells, membranes, and flexible multibody continua); (2) studies published primarily between 2018 and 2026, supplemented by foundational benchmark papers where necessary for theoretical continuity; (3) publications written in English; and (4) studies addressing linear or nonlinear dynamic phenomena, including free and forced vibration, wave propagation, stability, bifurcation, and transient response. Conversely, the exclusion criteria eliminated: (a) studies restricted to purely lumped-parameter or static structural analysis without a dynamic continuous-system component; (b) non-English records; (c) duplicate publications or extended abstracts lacking full methodological details; and (d) off-topic studies from unrelated disciplines.

Literature searches were conducted across major academic databases and publisher platforms—including Scopus, Web of Science, ScienceDirect, SpringerLink, Google Scholar, MDPI, and arXiv—and managed using Mendeley for deduplication and metadata organization. Search queries combined Boolean operators (*AND*, *OR*) across seven core methodological domains, as detailed in Table 1, and were supplemented by backward and forward citation tracking (snowballing).

Table 1. Core Search-Term Matrix Used to Organize the Literature Search Across Methodological Domains

No.	Methodological Domain	Core Search Concepts and Keywords
1	Classical Analytical Methods	continuous-system dynamics; analytical solution; separation of variables; beam/plate/shell vibration; eigenvalue problem
2	Approximate / Semi-Analytical Methods	continuous system; Galerkin truncation; Rayleigh–Ritz; perturbation; method of multiple scales; nonlinear vibration; resonance
3	Conventional Numerical Discretization	finite element method; finite difference; finite volume; boundary element; co-rotational FEM; nonlinear structural dynamics
4	Advanced / High-Order Numerical Methods	spectral method; spectral element method; hybridized discontinuous Galerkin; isogeometric analysis; high-order dynamics
5	Model-Order Reduction (MOR)	modal reduction; proper orthogonal decomposition (POD); reduced basis; component mode synthesis; spectral submanifold; quadratic manifold
6	Scientific Machine Learning (SciML)	scientific machine learning; SINDy; Weak SINDy; physics-informed neural networks (PINNs); data-driven elastodynamics
7	Neural Operators	neural operator; Fourier Neural Operator (FNO); DeepONet; Wavelet Neural Operator (WNO); Laplace Neural Operator (LNO)

Study selection followed a two-stage screening procedure in line with PRISMA 2020 (Page et al., 2021). First, titles and abstracts of all deduplicated records were screened to remove clearly irrelevant studies. Second, the full texts of remaining records were retrieved and evaluated against the eligibility criteria. For each included study, structured data extraction recorded the author(s), publication year, publication venue, continuous structural system examined, governing mathematical formulation, computational or analytical method, key performance metrics (accuracy, convergence, and computational speedup), and reported limitations. Quality assessment evaluated the mathematical rigor of the formulation, verification against analytical benchmarks or experimental data, and clarity of computational performance reporting. Given the methodological diversity spanning analytical, numerical, and operator-learning paradigms, a narrative and thematic synthesis accompanied by comparative matrices was conducted.

3. RESULTS AND DISCUSSION

A. Study Selection and Methodological Characteristics

Database queries identified 1,847 records across Scopus (n = 512), Web of Science (n = 398), Google Scholar (n = 623), and ScienceDirect (n = 314), complemented by 42 additional records identified via arXiv (n = 31) and citation tracking (n = 11). After removing 676 duplicates, 1,213 unique records were screened by title and abstract, leading to the exclusion of 891 records. Of the 322 reports sought for full-text retrieval, 29 could not be retrieved, leaving 293 full-text reports assessed for eligibility. Following full-text evaluation, 235 reports were excluded (112 not relevant to continuous-system dynamics, 38 published outside the target scope without foundational benchmark status, 6 non-English, and 79 restricted to static analysis). Ultimately, 58 studies were retained for qualitative synthesis. The complete PRISMA 2020 selection flow is illustrated in Figure 1.

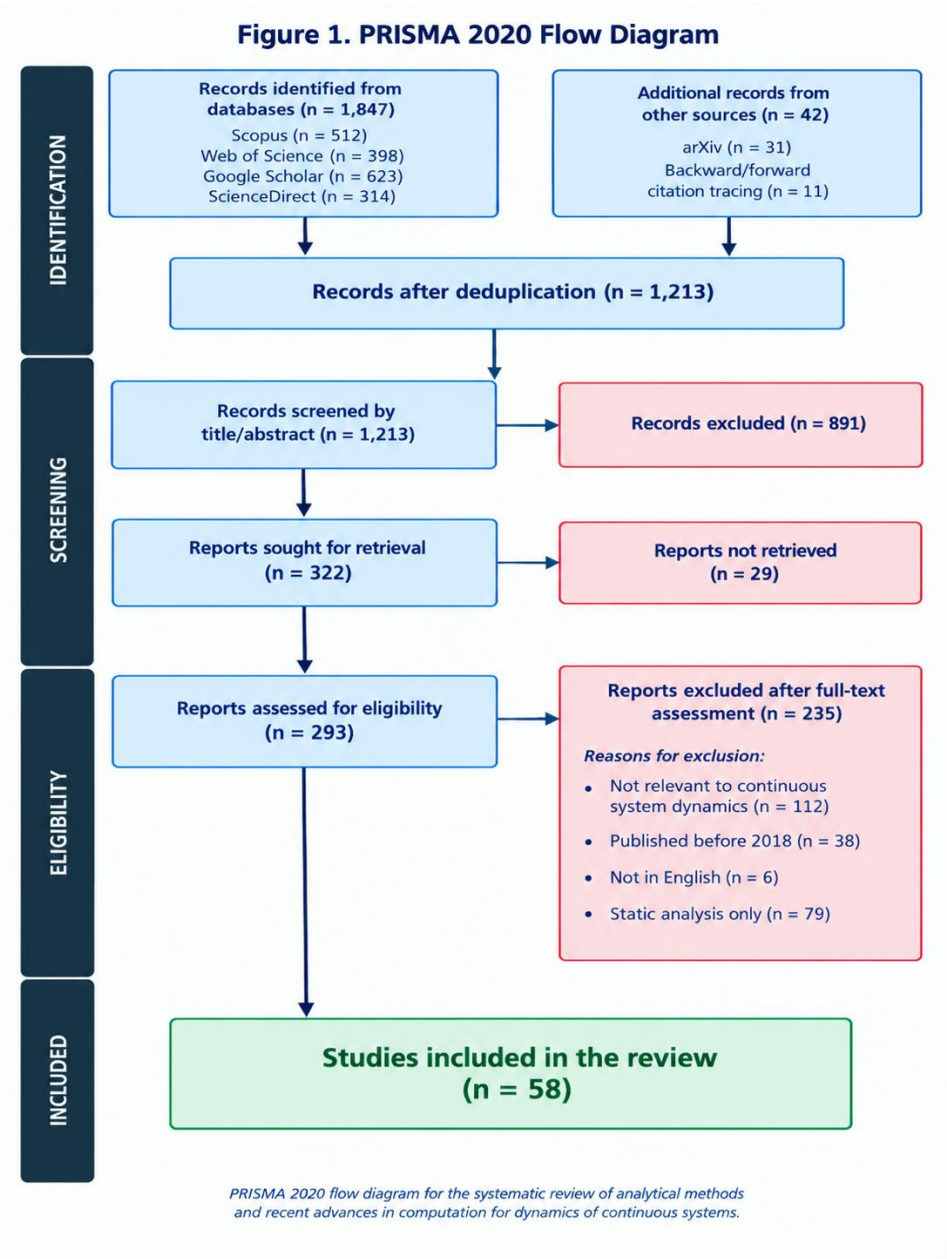


Figure 1. PRISMA 2020 Flow Diagram Showing Identification, Screening, Eligibility Assessment, and Inclusion of Studies

The 58 included studies were published across 28 peer-reviewed journals and proceedings—most prominently *Nonlinear Dynamics*, *Computer Methods in Applied Mechanics and Engineering*, *Journal of Sound and Vibration*, *Mechanical Systems and Signal Processing*, *International Journal of Mechanical Sciences*, and *Nature Machine Intelligence*. As summarized in Table 2, the included studies are balanced across five primary methodological categories: analytical and semi-analytical methods (22.4%), finite element and

discretization-based methods (17.2%), isogeometric and spectral high-order methods (20.7%), reduced-order modeling via spectral submanifolds and nonlinear manifolds (20.7%), and machine learning and neural-operator approaches (19.0%). Table 3 highlights a representative subset of ten landmark studies across these categories, summarizing their target structural systems, methods, primary focus, performance, and limitations.

Table 2. Distribution of Included Studies by Methodological Category

No.	Methodological Category	Number of Studies (n)	Percentage (%)
1	Analytical and Semi-Analytical Methods	13	22.4%
2	Finite Element and Discretization-Based Methods	10	17.2%
3	Isogeometric and Spectral High-Order Methods	12	20.7%
4	Reduced-Order Modeling via Spectral Submanifolds & Nonlinear MOR	12	20.7%
5	Machine Learning, SciML, and Neural-Operator Approaches	11	19.0%
Total	All Included Studies	58	100.0%

Table 3. Representative Data-Extraction Matrix for Landmark Studies Discussed in the Review

Study	System Studied	Methodological Approach	Main Focus	Reported Performance	Key Limitation
Meesala & Hajj (2019)	Cantilever beam with tip mass	Method of multiple scales	Nonlinear boundary conditions	Accurately captures nonlinear resonance response	Problem-specific analytical derivation
H.-Y. Chen et al. (2020)	Sandwich beam on nonlinear foundation	Galerkin truncation	Truncation order convergence	Establishes natural-frequency convergence criteria	Sensitive to retained truncation order
Ponsioen et al. (2020)	Finite-element beam models	Spectral submanifold (SSM) reduction	Forced-response curve computation	Substantial speedup up to 10,000 DOFs	Requires specialized manifold implementation
Kim & Lee (2021)	Rectangular Mindlin plate	DFT-based spectral element method	Frequency-dependent dynamic shape functions	High accuracy for wave propagation and vibration	Restricted to regular plate geometries
Kan et al. (2021)	3D nonlinear continuum structures	Co-rotational FEM with direct force correction	Geometrically nonlinear analysis	Eliminates nonlinear projector matrix; high efficiency	Subject to co-rotational small-strain assumptions
Jain & Haller (2022)	High-dimensional FE wing/beam structures	Direct invariant manifold / SSM computation	Nonlinear model-order reduction	Scales effectively to >100,000 FE degrees of freedom	Computationally specialized offline stage
Song et al. (2022)	Flexible multibody systems	Hybrid PSD–POD model reduction	Impact and frictional contact dynamics	Large computational savings for nonsmooth dynamics	Requires separate smooth/nonsmooth partitioning
Cenedese et al. (2022)	Beam oscillations, wake, and sloshing	Data-driven spectral submanifolds	Learning sparse reduced nonlinear models	Accurate prediction from trajectory data alone	Requires representative training trajectories
Geelen et al. (2023)	High-dimensional nonlinear dynamics	Quadratic manifold + Operator Inference	Non-intrusive nonlinear MOR	Constructs efficient reduced models without source code	Dependent on snapshot quality and scaling
Cao et al. (2024)	Parameterized differential equations	Laplace Neural Operator (LNO)	Transient and non-periodic operator learning	Rapid inference for non-periodic transient responses	Generalization bounded by training distribution

B. Classical Analytical Methods for Continuous-System Dynamics

Classical analytical formulations remain the theoretical cornerstone of continuous-system dynamics, led by the method of separation of variables, which decomposes governing partial differential equations into coupled spatial eigenvalue problems and temporal harmonic equations. This approach yields exact closed-form solutions for the free vibration of uniform beams, membranes, plates, and shells under classical boundary

conditions. Xing et al. (2022) conducted a comprehensive assessment of analytical solution methods for rectangular plate eigenvalue problems, evaluating the mathematical structure, exactness, and boundary-condition limitations of Levy-type, Navier-type, and symplectic dual-variable separation methods.

Building upon earlier analytical strip and trigonometric-series formulations—such as the work of Harik et al. (1992) on rectangular plates with intermediate flexible or rigid supports—Yessenbayeva et al. (2018) demonstrated the enduring relevance of separation-of-variables and series-expansion techniques for membrane and plate vibration problems under general boundary conditions. Although exact analytical methods contract sharply in applicability when confronted with irregular domains, material anisotropy, or strong nonlinearities, they remain indispensable for uncovering fundamental parameter dependencies and verifying numerical discretizations.

C. Approximate and Semi-Analytical Methods for Nonlinear Continuous Systems

Where exact closed-form solutions are unattainable, approximate analytical and semi-analytical techniques—principally the Rayleigh–Ritz method, Galerkin projection, perturbation expansions, and the method of multiple scales—extend analytical tractability to weakly nonlinear and coupled continuous systems. Meesala and Hajj (2019) applied the method of multiple scales directly to the governing partial differential equation and nonlinear boundary conditions of a cantilever beam carrying a tip mass, demonstrating that direct continuous-system perturbation captures subtle amplitude–frequency and boundary-nonlinearity effects that premature spatial discretization can obscure.

In parallel, H.-Y. Chen et al. (2020) investigated the convergence of Galerkin modal truncation for the nonlinear dynamic response of sandwich beams resting on nonlinear elastic foundations. Their study proved that retaining an insufficient number of linear trial modes leads to qualitative errors in predicted resonance responses and established quantitative criteria for selecting a convergent truncation order based on the natural frequency spectrum of the linearized system. Collectively, these semi-analytical methods preserve physical interpretability while bridging classical continuum analysis and full numerical discretization.

D. Conventional Numerical Discretization Methods

Conventional numerical discretization techniques—including the finite element method (FEM), finite difference method (FDM), finite volume method (FVM), and boundary element method (BEM)—remain the primary tools for modeling continuous systems with arbitrary geometry and complex boundary conditions (Jacinto Portela et al., 2022; Fu et al., 2023). While domain-discretization schemes (FDM, FVM, FEM) partition the entire structural volume, boundary-type formulations such as BEM and the singular boundary method reduce discretization to the bounding surface, lowering mesh-generation overhead for linear homogeneous media (Fu et al., 2023). In the temporal domain, specialized discrete-event quantized state system (QSS) integrators have also been advanced to handle stiff and discontinuous continuous-time dynamics efficiently (Castro et al., 2024).

Recent finite element research has focused heavily on capturing large-amplitude geometric and material nonlinearities while preserving numerical stability and efficiency. Khaniki et al. (2022) reviewed nonlinear dynamic formulations for hyperelastic beams, plates, and shells, highlighting advances in large-strain continuum mechanics. Müller and Bischoff (2022) formulated a consistent geometrically nonlinear Reissner–Mindlin shell finite element capable of robust large-rotation shell analysis, while Patuelli et al. (2024) developed and experimentally validated a nonlinear beam finite element incorporating bending–torsion coupling. To ensure long-time numerical stability, Brugnoli et al. (2025) introduced a linearly implicit energy–momentum preserving time-integration scheme based on non-canonical Hamiltonian formulations for geometrically nonlinear continuum mechanics. Furthermore, Kan et al. (2021) proposed a general co-rotational finite element framework based on direct internal force correction—eliminating the complex nonlinear projector matrix—which Zhou et al. (2026) subsequently extended into a rotation-dispersion-guided adaptive component-level co-rotational formulation for three-dimensional flexible multibody dynamics. Complementary nonsmooth contact and friction formulations (Song et al., 2020) further expand FEM capabilities to unilateral impact problems.

E. Advanced Numerical and High-Order Computational Methods

To overcome the dispersion errors and high degree-of-freedom counts of low-order finite elements in wave propagation and high-frequency vibration problems, high-order computational methods have been extensively developed. Spectral and spectral-element methods employ high-degree orthogonal polynomial or frequency-domain basis functions to achieve exponential convergence for smooth solutions (Hafeez & Krawczuk, 2023). Kim and Lee (2021) formulated a temporal- and spatial-domain discrete Fourier transform (DFT)-based spectral element model for rectangular Mindlin plates, deriving frequency-dependent dynamic shape functions that capture wave propagation and transient responses with minimal elements. Eisenträger et al. (2023) examined the numerical conditioning and dispersion properties of high-order spectral Euler–Bernoulli beam elements, while Gervasio et al. (2020) demonstrated through a rigorous computational

comparison that spectral element methods and NURBS-based isogeometric analysis (IGA) share closely related spectral and convergence properties under the Galerkin framework.

In parallel, discontinuous Galerkin (DG) formulations—which permit inter-element solution discontinuities linked via numerical fluxes—provide block-diagonal mass matrices well suited for explicit wave propagation and local time-stepping. Badrkhani et al. (2023) developed a matrix-free macro-element hybridized discontinuous Galerkin (HDG) method that avoids explicit global matrix assembly, significantly reducing memory bottlenecks for large-scale elastodynamics. Simultaneously, isogeometric analysis (IGA) unifies computer-aided design (CAD) geometry and finite element analysis by employing Non-Uniform Rational B-Splines (NURBS) as both geometric and solution basis functions, delivering exact geometric representation, higher inter-element continuity, and superior spectral accuracy across adaptive multi-patch domains (Buffa et al., 2022; Ren & Lin, 2023; Takacs, 2023).

F. Model-Order Reduction and Nonlinear Invariant Manifolds

When high-fidelity finite element models must be evaluated repeatedly across parameter sweeps, optimization loops, or real-time control, model-order reduction (MOR) becomes essential (Agathos et al., 2024). Classical projection-based MOR techniques include linear modal truncation, Proper Orthogonal Decomposition (POD) constructed from response snapshots (K. Lu et al., 2019), and Component Mode Synthesis (CMS) based on Craig–Bampton substructuring, where recent multilevel and interface-reduction strategies have substantially compressed interface degrees of freedom in large structural assemblies (Wu et al., 2018; Bondsman et al., 2024; Bui et al., 2024). However, linear subspace projections fail to capture modal coupling and invariant-manifold curvature in geometrically nonlinear continuous systems (Mazzilli et al., 2022).

Consequently, a major breakthrough during the review period has been the maturation of rigorous nonlinear model-order reduction via Spectral Submanifolds (SSMs) and Invariant Spectral Foliations (ISFs) (Szalai, 2020; Vizzaccaro et al., 2022). Defined as the smoothest invariant manifolds tangent to linear spectral subspaces at an equilibrium point, SSMs rigorously generalize nonlinear normal modes to dissipative forced systems. Ponsioen et al. (2020) and Jain and Haller (2022) developed automated direct parameterization algorithms capable of computing SSMs and forced-response curves directly from finite element models exceeding 100,000 degrees of freedom. This framework was extended to capture internal resonances, bifurcations, and quasi-periodic responses (Li et al., 2022; Li & Haller, 2022) as well as piecewise-smooth nonsmooth mechanical systems (Bettini et al., 2024). Crucially, Cenedese et al. (2022) introduced data-driven SSMs that learn sparse reduced-order nonlinear differential equations directly from experimental or simulation time-series data. Complementing SSMs, non-intrusive quadratic-manifold operator inference (Geelen et al., 2023), simulation-free modal-derivative reductions (Andersson et al., 2023), and hybrid proper symplectic–orthogonal decomposition schemes for flexible multibody contact dynamics (Song et al., 2022) provide computationally efficient pathways for industrial-scale nonlinear reduction.

G. Scientific Machine Learning and Physics-Informed Neural Networks

The integration of continuum mechanics with machine learning has given rise to Scientific Machine Learning (SciML), which embeds physical conservation laws and structural priors into data-driven architectures. In equation discovery, Kaheman et al. (2020) developed SINDy-PI (parallel implicit Sparse Identification of Nonlinear Dynamics) to recover rational and implicit governing equations, while Messenger and Bortz (2021) formulated Weak SINDy for PDEs, utilizing variational weak forms to identify governing partial differential equations accurately from highly noisy measurements. Similarly, Z. Chen et al. (2021) combined physics-informed neural networks with sparse regression to extract governing PDEs from scarce data, and Dong et al. (2024) embedded kernel functions into autoencoders to simultaneously discover low-dimensional latent coordinates and parsimonious nonlinear governing equations.

Concurrently, Physics-Informed Neural Networks (PINNs), formalized by Raissi et al. (2019), incorporate PDE residuals and boundary conditions directly into the neural network loss function, enabling mesh-free forward and inverse analysis. Rao et al. (2021) demonstrated that physics-informed deep learning can resolve two- and three-dimensional computational elastodynamic displacement and stress fields without labeled training data by enforcing momentum balance and constitutive laws. Nevertheless, applying PINNs to high-frequency vibration and wave-propagation problems in continuous systems frequently encounters severe training stiffness and unbalanced backpropagated gradients; Wang, Teng, et al. (2021) analyzed these gradient flow pathologies and introduced adaptive loss-balancing algorithms to stabilize convergence.

H. Neural Operators and Hybrid Operator-Learning Paradigms

Whereas standard PINNs approximate the solution for a single fixed instance of boundary conditions or material parameters, neural operators learn mappings between infinite-dimensional function spaces, enabling a single trained network to evaluate solution operators across entire families of parameterized PDEs with zero-shot discretization invariance (Azizzadenesheli et al., 2024). Li et al. (2021) introduced the Fourier

Neural Operator (FNO), which parameterizes integral kernel operators in Fourier space to achieve orders-of-magnitude computational speedups over traditional PDE solvers. To better capture localized, non-periodic, and transient structural dynamic responses where global Fourier bases struggle, Tripura and Chakraborty (2023) developed the Wavelet Neural Operator (WNO) for computational mechanics, and Cao et al. (2024) formulated the Laplace Neural Operator (LNO), which utilizes pole–residue formulations in the Laplace domain to accurately model transient and damped oscillatory systems.

In parallel, L. Lu et al. (2021) formulated DeepONet based on the universal approximation theorem for operators, employing a branch-and-trunk network architecture to encode input parameter functions and output spatiotemporal coordinates. A comprehensive benchmark across 16 PDE problems by L. Lu et al. (2022) demonstrated the complementary strengths of DeepONet and FNO under varying data regimes and complex geometries. Furthermore, hybrid physics-informed operator-learning architectures—such as Physics-Informed DeepONets (Wang, Wang, et al., 2021) and variational Physics-Informed DeepONets for brittle fracture and crack-path prediction (Goswami et al., 2022)—embed governing PDE residuals into operator training, reducing reliance on massive labeled simulation datasets while improving physical consistency.

I. Comparative Synthesis, Cross-Cutting Discussion, and Research Gaps

Across the 58 reviewed studies, a fundamental methodological trade-off emerges among physical interpretability, geometric and constitutive generality, offline training/setup overhead, and online repeat-query speed. Table 4 synthesizes these comparative characteristics across the seven methodological groups.

Table 4. Comparative Methodological Characteristics Identified Across the Reviewed Literature

Methodological Group	Accuracy & Fidelity	Computational Cost	Physical Interpretability	Repeat-Query Potential	Primary Limitation
Classical Analytical	Exact within idealized assumptions	Very low after closed-form derivation	Very high	Limited to simple parametric variations	Restricted to canonical geometries and linear regimes
Approximate / Semi-Analytical	High within perturbation/truncation range	Low to moderate	High	Moderate (analytic backbone curves)	Accuracy degrades for strong nonlinearities
Conventional Numerical (FEM/BEM)	High and broadly applicable	Moderate to very high at scale	High	Low for large-scale multi-query sweeps	High computational cost for fine meshes and long time
Advanced / High-Order (SEM/DG/IGA)	Very high; exponential convergence	Moderate	High to moderate	Good for high-precision wave/vibration runs	Higher implementation and multi-patch complexity
Model-Order Reduction (POD/SSM/OpInf)	High within retained manifold/subspace	Low online after offline construction	Moderate to high	Very strong for repeated forced/parametric runs	Offline computation and nonlinear manifold complexity
Scientific Machine Learning (PINNs/SINDy)	Problem- and loss-dependent	High training cost; low inference cost	Moderate (high for sparse SINDy)	Moderate (PINNs require retraining per instance)	Gradient pathologies and measurement noise sensitivity
Neural Operators (FNO/WNO/LNO/DeepONet)	Strong within trained parameter family	High offline training; ultra-low inference	Lower (black-box operator)	Exceptionally strong for parametric PDE families	Out-of-distribution generalization and data cost

As shown in Table 4, three overarching trends define the current research landscape. First, within physics-based numerical methods, structure-preserving time integration (Brugnoli et al., 2025) and adaptive co-rotational formulations (Kan et al., 2021; Zhou et al., 2026) demonstrate that substantial speedups and long-time stability can be achieved within the finite element framework by exploiting mathematical conservation laws and local reference frames. Second, nonlinear model-order reduction via spectral submanifolds and quadratic manifolds has bridged invariant-manifold dynamical systems theory with industrial-scale finite

element models (Jain & Haller, 2022; Cenedese et al., 2022; Geelen et al., 2023). Third, neural operators are evolving toward domain-tailored spectral kernels—such as Laplace and wavelet transforms (Tripura & Chakraborty, 2023; Cao et al., 2024)—that naturally accommodate the transient, non-periodic, and resonant dynamics of continuous structural systems.

Nevertheless, critical research gaps persist: (1) the absence of standardized, multi-paradigm benchmark problems that directly compare classical numerical solvers, nonlinear SSM/quadratic-manifold MOR, and physics-informed neural operators on identical large-scale structural dynamics problems; (2) limited verification of neural operators and data-driven MOR on complex multiphysics, contact-impact, and buckling-prone shell assemblies; and (3) insufficient uncertainty quantification and out-of-distribution physical guarantees for machine-learning surrogates in safety-critical engineering applications.

4. CONCLUSION

This PRISMA 2020-guided systematic review synthesized 58 studies examining analytical methods and recent computational advances for the dynamics of continuous systems, spanning classical exact solutions, approximate perturbation and Galerkin techniques, conventional and structure-preserving finite element methods, high-order spectral and isogeometric discretizations, nonlinear model-order reduction, physics-informed machine learning, and neural operators. The findings confirm that no single computational or analytical paradigm is universally superior across all continuous-system applications. Classical and semi-analytical methods remain indispensable for physical insight, resonance characterization, and solver verification, while high-order spectral, discontinuous Galerkin, and isogeometric formulations provide superior accuracy per degree of freedom for wave propagation and smooth structural dynamics.

For large-scale nonlinear and multi-query engineering problems, nonlinear model-order reduction via spectral submanifolds (SSMs), invariant spectral foliations, and non-intrusive quadratic manifolds offers a rigorous pathway to compress high-dimensional finite element models without losing essential nonlinear modal interactions. Concurrently, scientific machine learning (SINDy, PINNs) and neural operators (FNO, WNO, LNO, DeepONet) provide transformative speedups for inverse identification and parameterized PDE inference, particularly when hybridized with variational or residual-based physical constraints.

Based on the synthesized findings and identified research gaps, four strategic recommendations are proposed for future research in continuous-system dynamics: (1) establishing standardized open-source benchmark suites to evaluate analytical, high-order FEM/IGA, nonlinear MOR, and neural-operator methods on identical structural dynamics problems; (2) developing hybrid computational workflows that combine high-order or co-rotational finite element solvers with non-intrusive manifold reduction and physics-informed operator learning; (3) expanding rigorous uncertainty quantification and out-of-distribution physical conservation guarantees for SciML and neural-operator surrogates; and (4) scaling nonlinear invariant-manifold and operator-learning frameworks to coupled multiphysics and nonsmooth contact-impact shell and multibody systems. A summary of these recommendations is presented in Table 5.

Table 5. Summary of Strategic Recommendations for Continuous-System Dynamics Modeling

No.	Strategic Recommendation	Target Methodology / System	Expected Scientific and Engineering Impact
1	Establish standardized, multi-paradigm benchmark suites with open datasets and evaluation metrics.	Classical FEM/IGA, nonlinear MOR (SSM/OpInf), PINNs, and Neural Operators (FNO/WNO/LNO/DeepONet).	Enables objective, reproducible comparisons of accuracy, computational cost, and generalization.
2	Develop hybrid physics–data workflows integrating structure-preserving FEM with nonlinear MOR and neural operators.	Large-scale geometrically nonlinear beams, plates, shells, and flexible multibody systems.	Combines exact physical conservation laws with rapid multi-query surrogate inference.
3	Embed strict conservation laws and uncertainty quantification into SciML and neural-operator architectures.	Physics-informed DeepONets, Laplace/Wavelet neural operators, and weak-form equation discovery.	Enhances out-of-distribution reliability and certification readiness for safety-critical structures.
4	Extend data-driven spectral submanifolds and operator learning to coupled multiphysics and nonsmooth contact dynamics.	Fluid–structure interaction, thermo-mechanical shells, and frictional impact multibody assemblies.	Broadens the industrial applicability of reduced-order and operator-learning models.

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